On possible Θ vacua states in heavy ion collisions

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We have simulated the possible Θ vacua states in heavy ion collisions. In a quench like scenario, random phases of the chiral fields were evolved in a zero temperature potential incorporating the breaking of $U_A(1)$ symmetry. Initial random phases very quickly settles into oscillation around the values dictated by the potential. The simulation indicate that Θ vacua states that can be populated in heavy ion collisions is a coherent superposition of a number of modes.

QCD in the chiral limit ($m_u = m_d = m_s = 0$) possesses a $U_A(1)$ symmetry. Spontaneous breaking of the symmetry require a neutral pseudoscalar Goldstone boson with mass less than $\sqrt{3}m_\pi$, in addition to pion itself. However no such Goldstone boson is seen in nature. The problem was resolved by the discovery of non-perturbative effcets that violates the extra $U_A(1)$ symmetry. 't Hooft showed that because of the instanton solution of the Yang-Mills theory, the U(1) symmetry is not really a symmetry of the vacuum [1]. Term containing the so called vacuum angle (Θ) which breaks P and CP symmetry can be added to the Lagrangian. QCD requires very small $\Theta \sim 10^{-9}$, which explain the apparent P and CP symmetry in strong interaction. Dynamical breaking of the $U_A(1)$ symmetry is also obtained in large N (color) limit of the SU(N) gauge theory [2–4]. In this approach the dominant fluctuations are not semi-classical but of quantum nature.

Recently Kharzeev, Pisarski and Tytgat [5,6] argued that in heavy ion collisions non trivial Θ vacua states may be created. In the limit of large number of colors, the axial $U_A(1)$ symmetry of massless quarks may be restored at the the deconfining phase transition. As the system rolls back to confining phase, it may settles into a metastable state with non trivial Θ. The idea is similar to the formation of Disoriented chiral condensate (DCC) [7]. In DCC space-time region is created where the chiral condensate points in a direction different from that of the ground state. Similarly, in Θ vacua states space-time region with non-trivial Θ may be created.

After the suggestion of Kharzeev et al [5,6] several authors have looked into various aspects of non-trivial Θ vacua state that may be formed in heavy ion collisions [8–11]. Buckley et al [10,11] numerically simulated the formation of Θ vacua states, using the effective Lagrangian of Halperin and Zhitnitsky [12]. They assumed quench like scenario, rapid expansion of the fireball leave behind an effectively zero temperature region in the interior which is isolated from the true vacuum. Starting from an initial non-equilibrium state, they studied the evolution of phases of the chiral field. They saw formation of non-zero Θ vacuum within a time scale of 10^{-23} sec. In their formulation they used a dissipative term with friction constant γ. γ=200 MeV was chosen, which may be rather large as the fields evolve essentially in a zero temperature potential, where dissipation will be small. They also choose to ignore the Fluctuation-dissipation theorem which require that dissipation be associated with fluctuations (noise). They did not include such a term.

In the present paper we follow essentially the approach of Buckley et al [10,11] to investigate the possible creation of Θ vacua states in heavy ion collision, with some important differences. We use the effective Lagrangian developed by Witten [3] to discuss the $U_A(1)$ anamoly, and choose to omit the arbitrary dissipative term. In a quench like scenario, chiral phases are evolving in a zero temperature potential, where dissipative effects are supposed to be small. Also, neglecting dissipative term we are avoiding the problem of using a fluctuation term, non-inclusion of which will violate the fluctuation-dissipation theorem. We have also choose to use the proper time (τ) and rapidity (Y) as the most appropriate coordinate system for heavy ion collisions.

Effective non-linear sigma model which incorporates the breaking of $U_A(1)$ symmetry can be written as [3],

$$\mathcal{L} = f_\pi^2(\frac{1}{2}tr(\partial_\mu U^\dagger \partial_\mu U) + tr(M(U+U^\dagger)) - a(tr\ln U - \Theta)^2) \tag{1}$$

where U is a 3×3 unitary matrix with expansion, $U = U_0(1 + i\sum t^a\pi^a/f_\pi + O(\pi^2))$, U_0 being the vacuum expectation value of U, t^a are the generators of U(3) ($Trt^at^b = \delta^{ab}$) and π^a are the nonet of Goldstone boson fields. M is the quark mass matrix, which is positive, real and diagonal. We denote the diagonal entries as μ_i^2. They are the Goldstone boson sqared masses, if the anamoly term a (a/N in ref. [3]) were absent. Because M is diagonal U can be assumed to be diagonal , $U_{ij} = e^{i\phi_i}\delta_{ij}$.

In terms of ϕ_i's, the potential is,

$$V(\phi_i) = f_\pi^2(-\sum \mu_i^2 \cos\phi_i + a/2(\sum \phi_i - \Theta)^2) \tag{2}$$

It may be noted that as $\sum\phi_i$ arose from $tr\ln U$, it is defined modulo 2π. In the present work we use $\mu_1^2 = (114MeV)^2$, $\mu_2^2 = (161MeV)^2$,$\mu_3^2 = (687)^2$ and $a = (492MeV)^2$ [5,6]. With these parameters, the mass matrix in

(2) can be diagonalised to obtain $m_\pi^0 \sim 139$ MeV, $m_\eta \sim 501$ MeV and $m_{\eta'} \sim 983$ MeV, close to their experimental values.

Vacuum expectation values of the angles ϕ_i's can be obtained from the minimisation condition,

$$\mu_i^2 \sin\phi_i = a(\Theta - \sum \phi_j) \tag{3}$$

Solutions of this non-linear coupled equations has been discussed in detail by Witten [3]. If the equation has only one solution then physics will be analytic as a function of Θ. The solution vary periodically in Θ with periodicity 2π [3]. Also this solution must be CP conserving whenever CP is a symmetry of the equation [3]. However, it may happen that eq.3 has more than one solution. Then the solution are not CP conserving, rather a CP transformation exchanges them.

Existence of metastable states (Θ vacua states) can be argued as follows: the vacuum expectation values depend on the ratio a/μ_i^2, which in the mean field theory decreases with temperature $a/\mu_i^2 \sim (T_d - T)^{3/2}$, T_d being the decoupling temperature [6]. Then as the system rolls back towards the chiral symmetry breaking, a/μ_i^2 may be small enough to support metastable states. In ref. [5] it was shown that when $a/\mu_1^2 < .2467$ there is a metastable solution, which is unstable in π^0 direction unless $a < a_{cr}$, $a_{cr}/mu_1^2 \sim .2403$.

Appropriate coordinates for heavy ion scattering are the proper time (τ) and rapidity (Y). Assuming boost invariance, equation of motion for the phases ϕ_i's can be written as,

$$\frac{\partial^2\phi_i}{\partial\tau^2} + \frac{1}{\tau}\frac{\partial\phi_i}{\partial\tau} - \frac{\partial^2\phi_i}{\partial x^2} - \frac{\partial^2\phi_i}{\partial y^2} + \sum \mu_i^2 sin(\phi_i) - a(\sum\phi_i - \Theta)) = 0 \tag{4}$$

It is interesting to note that in this coordinate system, a dissipative term which decreases with (proper) time is effective. We have solved the coupled partial differential equations assuming a quench like scenario after the symmetry restoring phase transition. The rapid expansion of the high energy shell leaves behind an effectively zero temperature region, which is isolated from the true vacuum. The chirally symmetric fields then essentially evolves in the zero temperature potential.

The set of partial equations (4) were solved on a 64^2 lattice, with lattice spacing of $a = 1fm$ and time interval of $a/10$ fm. We also use periodic boundary condition. Solving eq.4 require field configuration at the initial time. We assume the initial time as τ_i=1 fm. Initial phases (ϕ_i and $\dot{\phi}_i$) were chosen according to following prescription,

$$\phi_i = R_N/(1 + exp((r - r_{cr})/\Gamma) \tag{5}$$
$$\dot{\phi}_i = R_N/(1 + exp((r - r_{cr})/\Gamma) \tag{6}$$

where R_N is a random number within the interval $[-2\pi/16, 2\pi/16]$. We use r_{cr}=20 fm and Γ=.5 fm.

The phases were evolved according to the eq.4 for arbitrarily long time 50 fm in the zero temperature potential. The idea is to see whether nontrivial Θ vacua emerge during the evolution. We have considered two situation, (i) Θ=0 and (ii)Θ=$4\pi/16$ within the radius r_{cr} and zero beyond that radius. For Θ=0 emergence of non-zero ϕ's will be signature of Θ vacua.

In fig. 1, the evolution of the spatially averaged phases are shown,

$$\phi_i = \frac{\int \phi_i(x,y)dxdy}{\int dxdy}, i = u, d, s \tag{7}$$

For Θ =0, the potential is minimised for $\phi_u = \phi_d = \phi_s = 0$. The averaged fields also fluctuate around zero througout the evolution. This shows that chirally symmetric fields donot evolve into a non-trivial Θ vacua. The situation is different for finite Θ. For $\Theta = 4\pi/16$ the potential is minimised for ϕ_u=0.502, ϕ_d=0.243 and ϕ_s=0.013. Chirally symmetric fields very quickly reaches these values and oscillate around it. The oscillation continues for long, with little damping. As will be shown later, the continues oscillation of the spatially averaged phases indicate that a large number of modes contribute to the Θ vacua state.

We define a correlation function,

$$C(r) = \frac{\sum_{i,j} \vec{\phi_i}.\vec{\phi_j}}{\sum_{i,j} |\vec{\phi_i}||\vec{\phi_j}|} \tag{8}$$

such that the distance between the lattice points i and j is r. $C(r)$ specifies how the three component $\vec{\phi} = (\phi_u, phi_d, \phi_s)$ at two lattice points are correlated. In fig.2, we have shown the evolution of the correlation function for Θ=0 and

Θ=0.785 at different times, τ=1,6,11,16 and 21 fm. Initially at τ=1 fm, for both the cases, correlation length is about 1 fm, the lattice spacing indicating the fact that initial fields were random, with no correlation. The correlation length increases at later times. But for Θ=0, the increase is marginal. However for finite Θ, correlation length increases very rapidly, and assumes quite large values. Physically, for finite Θ all the $\vec{\phi}$'s try to align themselves in some direction (i.e. in the Θ direction) thereby giving very large correlation length even when they are separated by a large distance. This figure clearly demonstrated the possibility of parity odd bubbles formation. Initially random phases evolve such that they points in the same direction, forming a large parity odd bubble.

To compare our results with that of Buckley et al [10], we have also studied the evolution of the different modes of the phases ϕ_i. At each alternate time step, we apply a fast fourier transform to the spatial data. The fourier transformed data are then integrated over the angles to obtain momentum distribution,

$$\phi_i(k) = \frac{1}{2\pi}\int_0^{2\pi} \phi_i(k,\theta) d\theta, i = u, d, s \tag{9}$$

Each mode was averaged over some narrow bin. In fig.3, we have shown the evolution of ϕ_u for Θ=0 (upper panel) and Θ=0.785 (lower panel). For both the cases, the modes oscillate throughout the evolution. Initially amplitude of the oscillation decreases, but at later time it remains more or less same. This is indicative of the fact that dissipative term decreases with time. We also find that higher modes are suppressed compared to zero mode, but the suppression is not as large as obtained by Buckley et al [10,11]. Even after a large interval of time, higher modes donot become negligibly small. For Θ=0, throughout the evolution, the zero mode as well as higher modes, oscillate about zero. But for finite value of $\Theta = 4\pi/16$ the modes oscillate about some definite nonzero value. For ϕ_d and ϕ_s, we have obtained qualitatively similar results. Indeed the evolution of different modes are in accordance with fig.1, where we have shown the evolution of spatial averaged phases. Spatially averaged phases were found to oscillate about some fixed value dictated by the minimum of the potential. Present simulation results are qualitatively different from the simulation results of Buckley et al [10,11]. They found that for finite Θ, the modes quickly reaches the value dictated by the Θ vacuum. The difference is essentially due to the strong dissipative term in the equation of motion of Buckley et al [10], which we have omitted. We have checked that if we include an arbitrary dissipative term, zero mode as well as higher modes quickly evolve into a constant value, dictated by the minimum of the potential. Also, in the simulation of Buckley et al [10,11], the Θ vacua is essentially consists of zero mode, while we find that in absence of dissipation, higher modes also contribute substantially. The Θ vacua is thus a coherent a superposition of a number of modes.

Present simulation indicate that in heavy ion collision non-trivial Θ-vacua state that is a coherent superposition of a number modes can be formed. What will be the signature of such a state. As such detecting Θ states are difficult. Kharzeev and Pisarski [6] estimated the P-odd observables are on the order of 10^{-3}, a small effect. Also as discussed by Voloshin [9] the so called signal of Θ states may be faked by "conventional" effect such as anisotropic flow etc. Whatever be the signal of the Θ vacua states, with a large number of modes contributing, signal will be broadened, effectively diluting the detection probability.

To summarise, we have simulated the non-linear sigma model incorporating the dynamical $U_A(1)$ breaking. Assuming boost invariance equation of motion of the chiral phases were solved on a 64^2 lattice with lattice spacing a=1 fm. It was shown that phases of chiral fields with finite Θ oscillate around the values dictated by the minimum of the potential. Correlation studies shows that initially uncorrelated phases very quickly develops large correlation length, indicating formation of a 'Parity Odd' bubbles. Fourier analysis of the modes indicate that the Θ vacua states are coherent superposition of a number of modes, which continue to oscillate with time.

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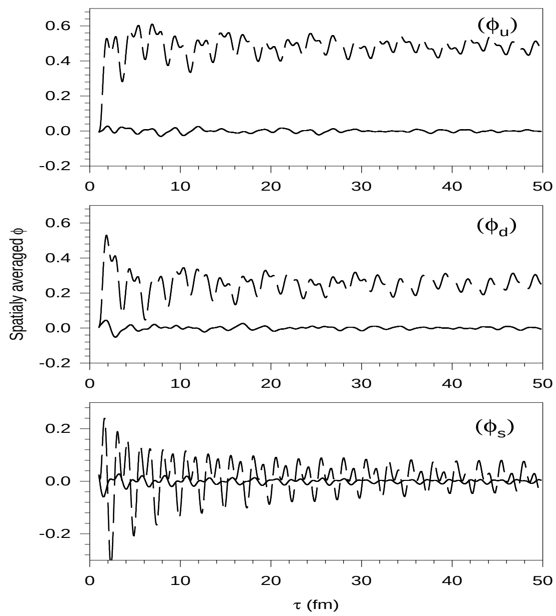

FIG. 1. Evolution of spatially averaged ϕ_i, i=u,d and s for Θ=0 (solid line) and Θ=0.785 (dashed line).

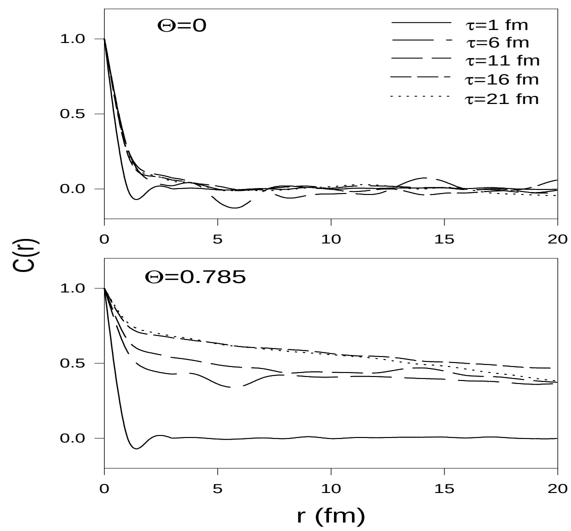

FIG. 2. Correlation function at different times for Θ=0 (upper panel) and Θ=0.785 (lower panel).

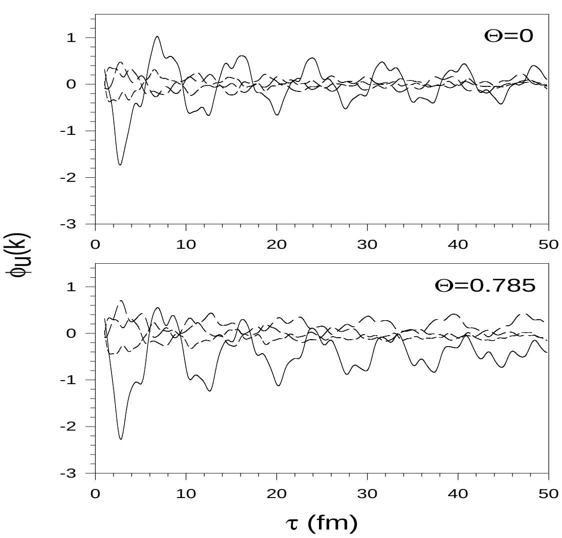

FIG. 3. Evolution of the Fourier transformed $\phi_u(k)$ with time for different modes, k=6.2, 18.5, 30.9, 43.3 and 55.5 MeV. The solid line is for the lowest mode.